E 1966 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2011

Third Semester

Core Course–3—CALCULUS

(Common for Model I and Model II B.Sc. Mathematics and B.Sc. Computer Applications)

Time : Three Hours Maximum Weight : 25

Part A (Objective Type Questions)

Answer all *questions.*
Each bunch of four questions carries weight 1.

I. 1 Value of $D^n(ax+b)^n$ is ———.

2 The Mclaurin's series for e^x is ———.

3 The curve $y = f(x)$ is ——— at $(c, f(c)$ if $f''(c) < 0$.

4 The curvature of a straight line at every point is ———.

II. 5 The evolute of a curve is the envelope of its ———.

6 Find $\frac{\partial f}{\partial x}$ if $f(x, y, z) = e^{-xyz}$.

7 If $W = f(x, y), x = g(r, s), y = h(r, s)$ then $\frac{\partial W}{\partial r} =$ ———.

8 Write the discriminant of the function $f(x, y)$.

III. 9 Let f be continuous on $[-a, a]$. If f is even then $\int_{-a}^{a} f(x)\,dx = \ldots$.

10 Find the volume of the solid of cross-sectional area $A(x) = 2x$ from $x = 0$ to $x = 2$.

11 Write the formula for the length of the curve $x = f(t), y = g(t), a \le t \le b$.

12 Write the surface area formula in the differential form.

IV. 13 Express the area bounded in the plane by the x-axis, the lines $y = x$ and $x = 1$, as a double integral.

14 Write the formula for the area of a bounded region R in the polar co-ordinate plane.

15 Define the volume of a closed bounded region D in space using triple integral.

16 Write the equation of a sphere of radius 5 with centre at origin, in spherical co-ordinates.

$(4 \times 1 = 4)$

Turn over

Part B (Short Answer Type Questions)

Answer any **five** *questions.*
Each question carries weight 1.

17 Differentiate the equation n times with respect to x: $\left(1-x^2\right)y_2 - xy_1 - y = 0$.

18 Find the asymptotes parallel to the co-ordinate axes of the curve $\left(x^2+y^2\right)x - ay^2 = 0$.

19 Find $\frac{dw}{dt}$ at $t = 0$ if $w = xy + z$, $x = \cos t$, $y = \sin t$, $z = t$.

20 Find the local extreme values of $f(x, y) = x^2 + y^2$.

21 Evaluate $\int_0^1 t^3\left(1+t^4\right)^8\, dt$.

22 Find the volume of the sphere generated by rotating the circle $x^2 + y^2 = 1$ about the x-axis.

23 Find the area of the region enclosed by the curve $y = x^2$ and the line $y = x + 2$, by double integrals.

24 Evaluate the cylindrical co-ordinate integral $\int_0^{2\pi}\int_0^2\int_0^{r^2} z\, dz\, r\, dr\, d\theta$.

$(5 \times 1 = 5)$

Part C (Short Essay Questions)

Answer any **four** *questions.*
Each question carries weight 2.

25 Find the points of inflexion on the curve $y = (\log x)^3$.

26 Find the evolute of the parabola $y^2 = 4ax$.

27 Find the greatest and smallest values of that the function $f(x, y) = xy$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$.

28 Find the length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 4$.

29 Find the area of the surface that is generated by revolving the portion of the curve $y = x^3$ between $x = 0$ and $x = 1$ about the x-axis.

30 Evaluate $\int_0^1 \int^{\sqrt{1-y^2}} xy\, dx\, dy$ by expressing it as an equivalent integral with the order of integration reversed.

$(4 \times 2 = 8)$

Part D (Essay Questions)

Answer any two questions.
Each question carries weight 4.

31 If $y = \dfrac{\sin^{-1} x}{\sqrt{1-x^2}}$, where $-1 < x < 1$ and $-\dfrac{\pi}{2} < \sin^{-1} x < \dfrac{\pi}{2}$, show that

$$\left(1-x^2\right)y_{n+1} - (2n+1)xy_n - n^2 y_{n-1} = 0.$$

Assuming that y can be expanded in ascending powers of x in the form $a_0 + a_1 x + \cdots + a_n x^n + \cdots$, prove that $(n+1)a_{n+1} = n a_{n-1}$, and hence obtain a general term of the expansion.

32 The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

33 Find the average value of $F(x, y, z) = \left(1 - x^2 - y^2 - z^2\right)^{-1/2}$ over the sphere $x^2 + y^2 + z^2 = 1$.

$(2 \times 4 = 8)$